# Review Article: A Manual for One User[1]

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*Tonality: An Owner's Manual*, by Dmitri Tymoczko. Oxford Studies in Music Theory. New York: Oxford Academic, 2023. xiii, 612 pp.

Dmitri Tymoczko (henceforth, "the author") dedicates his recent book, *Tonality: An Owner's Manual* (henceforth, "the book"), "*to anyone who cares enough to make it all the way through*" ([p. v], original emphasis). I assume that by "[caring] enough" the author means caring about *tonality*, the first word in the title of the book. By "[making] it all the way through," I assume that he refers not only to the book's extraordinary length, but also to its sprawling contents. "This book grew like a city" the author begins the "Preface and Acknowledgments" ([p. xi]), "each gleaming draft built over the ruins of a previous version." Referring to a last-moment revision made after he thought "the book was essentially finished…," the author goes on to confide (p. xii):

> This last revision, so drastic and so late, raised the disturbing possibility that I had ceased writing a book in any conventional sense: what had begun as a journey had turned into a way of life, my manuscript evolving into a repository for my latest thinking on all things music. Every time I learned something new, or changed my mind, I updated the relevant file on my computer. Having faced the prospect of spending the rest of my life endlessly writing and rewriting the same words—along with the attendant familial disapproval—I resolved to stop. The result is this snapshot of my thinking as of the time of publication. Already I worry that readers will be reluctant to climb so far out on my own little conceptual limb. And though I have tried to be clear, I am aware that the material is extremely challenging.

Given that the book is so very unconventional, a conventional review of it, in terms of both length and content, would correspondingly miss the mark. Indeed, the first *practical* decision I had to make, in deciding to review the book at all, was to do so in two parts: Part I (the present review) concentrating on roughly the first half of the book, chapters 1–5 (pp. 1–252), to be complemented by a Part II (a projected sequel) that will concentrate on the second half, chapters

[1] The review was solicited by *The Journal of the American Musicological Society*, but due to its length was prevented from being published there. Many thanks to Ehud de Shalit for commenting on an early draft of the mathematical appendix.

6–10 (pp. 253–532). Such a broad division of the book into two parts is suggested by the author when he outlines (p. 33):

> This book is framed by two largely analytical chapters. Chapter 2 argues that a variety of rock progressions, all unusual from the standpoint of classical theory, reflect straightforward features of musical geometry—and hence that the intuitive competence of the rock musician is in part a matter of knowing one's way around the space of chromatic triads. This chapter is meant to provide an accessible introduction to my general approach. Chapter 10 makes a broadly similar point about Beethoven, focusing on what I call the "Ludwig" schema, and considering some of the philosophical challenges his music poses. This is more of a culmination, linking technical issues in voice-leading geometry with philosophical questions about analysis. Together, the two chapters suggest that a range of different musical styles can be linked by a subterranean geometrical logic.

"The latter part of the book [i.e., chapters 6–10]," the author further clarifies, "turns to the *harmonic system*, a set of initially implicit norms that were eventually codified by figures such as Rameau and Riemann" (p. 34, original emphasis). As I will argue subsequently in this review, only in this part of the book is the term "tonality" engaged in any normative sense.

A second and more substantial decision that I had to make concerns the algebra that underlies the "geometrical logic" so central to the book, particularly its first part. After struggling in vain to make sense of the "spiral diagrams" that supposedly capture the logic, I came to an important conclusion: the diagrams suggest a valuable music-algebraic insight, provided that, as the author hopes in the dedication cited above, *one cares enough* to penetrate the "extremely challenging" content underneath which it is buried. For reasons to be clarified shortly I will use the rubric "Voice-Leading Sums under Translation and Tymoczkian Permutation of Chords"—VLS/TTPC for short, to refer to this insight, nowhere formulated in the book yet hinted at on more than one occasion.[2] Mathematically inclined readers will find in the appendix of this review its formalization and proof. Since, as I will argue in detail in due course, the spiral diagrams, "the main theoretical models used in this book" (p. 580), make hardly any sense except as poorly executed attempts to express aspects of VLS/TTPC, I have found it necessary to structure this review somewhat unorthodoxly.

Focusing on roughly the first half of the book (chapters 1–5), this review article consists of five main sections, grouped into two large parts. In sections 3–5 I challenge four claims central to the book at large. Specifically, in section 3 I challenge the claim that the "spiral diagrams" are "simple and intuitive" geometric representations of what the author refers to as "hierarchically nested

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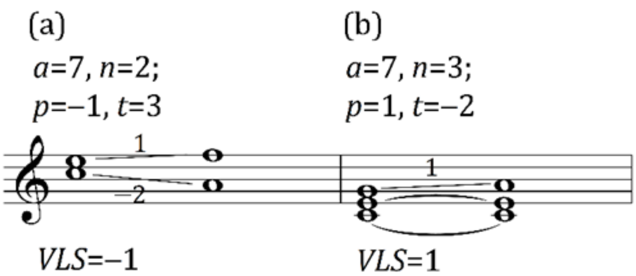
[2] Most explicitly on pp. 41–42, 68–69, 90, and 98.

transposition" (p. 42). In section 4, I challenge the claim that the author's idea of "transposition along a collection" is "the single most important concept in music theory" (p. 37). In this section I also challenge the claim made at the very outset of the book, in reference to the famous chordal succession that opens Gesualdo's 1611 madrigal, *Moro lasso*, that there is "a unified musical logic" (p. 9) by which, in certain selected passages "… Stravinsky, Mahanthappa, and Pachelbel [among others, like Mozart, Beethoven, Schubert, and Debussy] are all doing the same thing" (p. 6)—namely, the "trick" that Gesualdo supposedly performs in these measures. Finally, in section 5 I challenge the claim, implicit in the book's title, that its subject matter is tonality, a challenge that applies to roughly the first half of the book, on which this review focuses. Going backwards, in section 2 I prepare the arguments of sections 3−5 by placing the four critiqued claims, directly or obliquely, in the context of VLS/TTPC. Finally, in section 1 I present VLS/TTPC informally.

Again, for a review, this is admittedly an unusual structure. However, to have presented sections 3–5 independently of sections 1–2 would have been tantamount to dismissing the book, or at least its first half, as incoherent. I would like to believe, rather, that despite the book's severe shortcomings, a weakly articulated idea in it is worthy of attention.

## 1.

Consider Example 1. Relative to an underlying "scale" defined by an "octave" consisting of seven "steps" (hence, $a = 7$, $a$ being the number of steps to the octave), one sees two pairs of "chords" or *ordered* sets of steps: dyads in (a) and triads in (b) (hence, $n = 2$, 3, respectively, $n$ being the number of chordal elements). For present purposes, it will be convenient to refer to the steps by their conventional names, e.g., C5, E5. One should keep in mind, though, that since the underlying scale is "purely diatonic," certain distinctions implicit in staff notation and conventional note-nomenclature are irrelevant, notably the distinction between intervallic *qualities*: e.g., "*major* second" versus "*minor* second."




**Example 1** Permutation and translation of a dyad (a) and a triad (b) relative to the 7-scale

The parameters $p$ and $t$ given in each of the two parts of the example refer respectively to the operations of cyclic permutation and translation by which the second chord is derived from the first. Given an $n$-ad (i.e., dyad, triad, tetrad, etc., for $n = 2, 3, 4$, etc., respectively), its "translation" is the transposition of each of its $n$ steps by some fixed step-interval (e.g., "descending fifth"). As for "cyclic permutation," it corresponds, approximately, to "inversion" in the sense of elementary music theory: "first inversion," "second inversion," etc. I say "approximately" because the correspondence only works under the assumptions that the chordal elements lie within a single octave ("closed position") and are ordered from low to high. Following the author, subsequently in this review I will point out that this limited sense of "permutation-as-elementary-theory-inversion" may be generalized to chords in any spacing and ordering. Turning back to Example 1, one may easily verify that the second dyad in (a) is the combined result of one downward permutation of the first dyad (hence, $p = -1$) and a translation by "ascending fourth" (hence, $t = 3$; the order in which one performs these two operations is irrelevant). Similarly, the second triad in (b) is a translation of the first triad by –2 ("descending third") combined with one upward permutation (i.e., "root position" to "first inversion").[3]

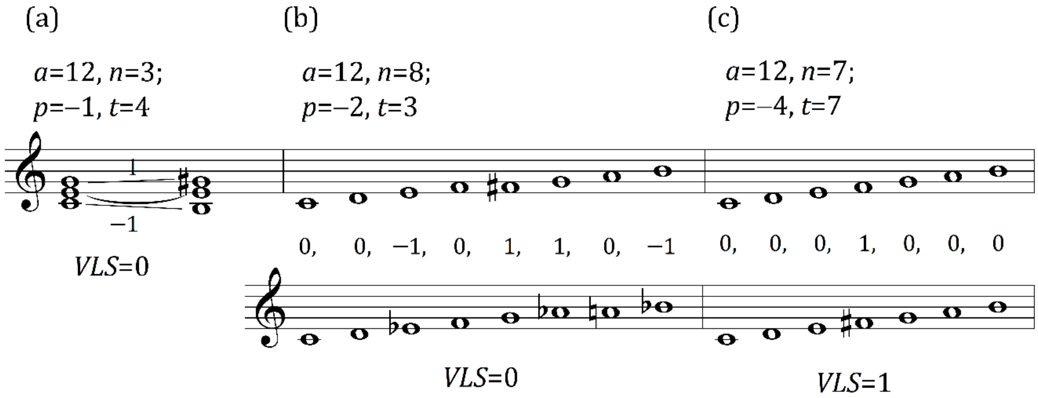
Before looking at the remaining notations in the example, consider another set of examples, analogous to Example 1, except that the underlying scale is now defined by a 12-step octave (i.e., $a = 12$), rather than seven (Example 2). Since staff notation is inherently incapable of representing the intervals of the 12-step scale short of interpreting them arbitrarily in 7-step fashion, as in "augmented *prime*," "minor *second*," "doubly diminished *third*," etc., all for the unit interval, my earlier caveat concerning staff notation is even more pertinent. Therefore, in referring to the steps of this "purely chromatic" scale in the text, I will avoid conventional nomenclature altogether and will use instead numbers as familiar from musical "set theory."[4] Thus, in Example 2(a) we have a $(0, 4, 7)$ triad that is permutated downward once ($p = -1$) and translated four steps upwards ($t = 4$), and in Example 2(b) an octad ($n = 8$), represented "melodically" on the upper staff (that is, ordered in time), is translated by 3 and permutated by –2 (lower staff).[5] Finally, in Example 2(c), the septad

[3] Our atomic objects are "steps" rather than "step *classes*," and therefore permutation in the sense discussed is not truly "cyclic": a succession of permutations in one direction can only yield an *octave translation* of the original chord, never the original chord. Readers uncomfortable with the term "cyclic permutation" may wish to replace it mentally with "quasi-cyclic permutation." Henceforth I will often write "permutation" in the sense of "(quasi-) cyclic permutation."

[4] Throughout this review, "number" refers to integers, that is, whole numbers.

[5] In terms of applying the same operation recursively, permutation behaves exactly like translation. Thus, $p = -2$ means that permutation by $-1$ (one descending permutation) is applied twice in succession.

$(0,2,4,5,7,9,11)$, also represented melodically on the upper staff, is permutated and translated by $(p,t) = (-4,7)$ (lower staff).




**Example 2** Permutation and translation of a triad (a), octad (b), and septad (c), all relative to the 12-scale

In Example 1 as well as Example 2(a), slurs or line segments connect elements of the first chord with corresponding elements of the second. Slurs represent an identity between the two corresponding elements. That is, the interval, from the slurred element of the first chord to its corresponding element in the second (the value of which is obtained, as usual, by subtracting the former from the latter), is zero. Lines represent non-zero intervals, the values of which are given above or below the corresponding lines. In Examples 2(b) and 2(c), where chords are represented "melodically," intervals between corresponding chord-elements, be they zero or non-zero, are displayed in the proper positions *between* the staves.

For any ordered pair of chords of the same cardinality $n$, an ordered set of $n$ numbers of the type just described may be thought of as the "voice leading" from the first chord to the second. Very differently from permutation and translation, the voice leading "generates" the second chord from the first in the manner of "part writing": "displace the first step of the first chord by the first voice-leading interval, displace the second step by the second interval," etc.

Now, the *sum* of the $n$ intervals that constitute the voice leading from the first chord to the second may be thought of as a measure of how "balanced" the voice leading is: the smaller the *absolute value* of the sum, the more balanced the voice leading (trivially, the most balanced voice leading is where the second chord is identical to the first). Somewhat like vectors, in other words, exerting in opposite directions different magnitudes of force on the same object, the $n$ "voices" express an "overall force" the magnitude and direction of which corresponds to their sum. The priorly mentioned VLS/TTPC insight states that, given an $a$-scale and two $n$-ads relative to this scale, the second of which is generated from the first by permutation $p$ and translation $t$, the corresponding voice-leading sum, or $VLS$, equals ($a$ times $p$) plus ($n$ times $t$). In mathematical notation:

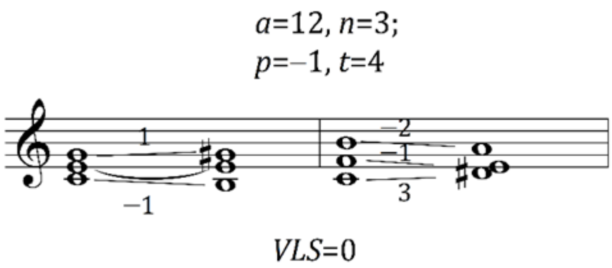
$$VLS = ap + nt. \tag{1}$$

Even though, as we shall see in section 3, the author employs an idea equivalent to and, depending on algebraic context, even the same as $VLS$ (vaguely conceived as a measure of voice-leading balance), and even though, in the context of this idea, he manipulates chords using parameters equivalent to $p$ and $t$, Equation 1, which the reader may readily check is satisfied in all five cases of Examples 1 and 2, is never stated in the book, let alone proven.

## 2.

What is striking about the VLS/TTPC insight as encapsulated in Equation 1 is that given an $a$-scale and an $n$-ad of interest, $VLS$ depends only on $p$ and $t$, that is, the values by which the $n$-ad is permutated and translated, respectively. As the author notes on pp. 68–69 (though in a specific algebraic context, as discussed further in section 3), $VLS$ does *not* depend on the *type* of $n$-ad, where "type" refers to a class containing all and only $n$-ads relatable to each other by translation and permutation.[6] Thus, for example, *any* 7-scale dyad, translated and permutated, as in Example 1(a), by $p = -1$ and $t = 3$, satisfies $VLS = -1$; similarly, any 12-scale triad, permutated and translated, as in Example 2(a), by $(p, t) = (-1, 4)$, satisfies $VLS = 0$. Example 3 displays representatives of two different types of 12-scale triads, the permutation and translation of either of which by $(p, t) = (-1, 4)$ satisfies $VLS = 0$. Note in Example 3 how different the two *voice leadings* are, unlike their sums.

[6] An $n$-ad of interest, e.g., the 12-scale triad $(0, 4, 7)$, is usually studied as a representative of an $n$-ad *type* of interest, that is, the class of all and only $n$-ads relatable to the $n$-ad of interest by permutation and translation.




**Example 3** Permutation and translation by the same values of two different types of 12-scale triads

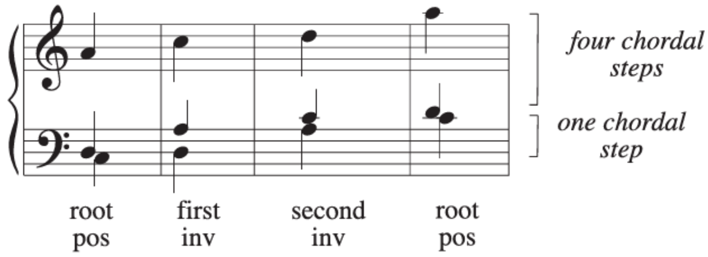
At the same time, it is important to bear in mind the insight's inherent limitations. Most notably, VLS/TTPC only concerns $n$-ads of the same type, that is, $n$-ads that are relatable by permutation and translation. Since, for example, such 12-scale triads as (0, 3, 7) and (0, 4, 7) are *not* of the same type, VLS/TTPC has nothing to say regarding "progressions" that include representatives of both. I note this built-in limitation of VLS/TTPC because the author seems to invest considerable efforts in the book to "work around" it. Such misguided efforts inevitably lead, as will be discussed in subsequent sections of this review (especially the latter part of section 4, concerning "Gesualdo's trick"), to content that ranges from the deeply problematic to the flatly incoherent.[7]

First, however, two central concerns in the book should be mentioned, as they, too, are best understood vis-à-vis VLS/TTPC.

The first concern is connected to our working assumption that chords are in closed position and are ordered from low to high. Since at least the restriction to closed position is quite reasonably *not* assumed in the book, the notion of "(quasi-cyclical) permutation," thus far assumed to correspond to "inversion" in the sense of elementary music theory, must be generalized to chords in any spacing. (Regarding order, see the next paragraph.) For, given an arbitrarily spaced chord, $p = 1$ cannot simply mean that the lowest (and first) element of the chord is octave-displaced to the first available position above its highest (and last) element, in analogy to "inversion" in the usual, elementary-theory sense. Rather, as discussed by the author on pp. 37–38 and shown in the left-hand portion of Figure P2.6 (p. 40), reproduced here as Example 4, the "inversion" must preserve not only the (quasi-) cyclical ordering of the original chord, but also *its spacing*. I will refer to this generalized sense of chordal

[7] An additional limitation of VLS/TTPC reflects the limited adequacy of each of the two scales that interest the author the most, the "purely diatonic" 7-scale and the "purely chromatic" 12-scale, one considered apart from the other, as models of music that might be termed "tonal"—a term that appears in the nominal form in the title of the book. This limitation, already hinted at in remarks concerning staff notation and conventional note (and interval) nomenclature, will be revisited in section 5.

"inversion," captured formally in the appendix of this review in Definition 2.2(B), as "Tymoczkian permutation," though I shall often abbreviate the term to just "permutation." The theorem encapsulated in Equation 1 is proven in the appendix relative to "permutation" in this sense.




**Example 4** From the book's Figure P2.6 (p. 40). "Forming registral inversions by moving each voice along the intrinsic scale formed by the chord's notes." © Oxford University Press 2023. Reproduced with permission of Oxford Publishing Limited through PLSclear

It is important to note that, for reasons that need not concern us here, Tymoczkian permutation is conceived in the book as "transposition along a collection." I will revisit the notion of Tymoczkian permutation in section 4, where vis-à-vis the author's analytical misusage of it, its dependence on the *ordering* of chordal elements will be made apparent.

If Tymoczkian permutation in the book is slightly misused and vastly overrated ("the single most important concept in music theory," p. 37), a second major concern in it, the "spiral diagrams," is not so much overrated by the author as its execution, discussed in detail in section 3 of this review, is so idiosyncratic, as to defy norms of rational discourse. In an attempt to make sense of these diagrams nonetheless, based on hints that the author provides here and there (some of which were already noted), I will tentatively interpret the claim on p. 42—that they represent "hierarchically nested transposition"—to mean that they represent *recursive* translation and (Tymoczkian) permutation of an $n$-ad of interest relative to an $a$-scale of interest (usually, the 7-scale or the 12-scale), generating what may informally be described as a "harmonic sequence."[8] That is to say, some *fixed* pair $p$ and $t$ (respectively representing the degrees of permutation and translation), accounts for the relationship between pairs of successive $n$-ads in the recursion. Through the remainder of this section, I will outline a thought-process that, under this interpretation, could have guided a quest for representing geometrically such recursions.

To the extent that cases of relatively balanced voice leading seem inherently attractive, a reasonable as well as practical strategy in translating

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[8] See chapter 4, "Repetition" (pp. 155–202), and its "Prelude: Sequence and Function" (pp. 151–54).

Equation 1 into musically relevant configurations is to restrict the absolute value of $VLS$. Studying the expression

$$ap + nt,$$

it is not difficult to see that, since $a$ and $n$ are both positive (specifically, $a > n \geq 2$), if the expression's value lies strictly in between $-(a + n)$ and $a + n$, then, unless $p$ or $t$ equal zero, these two parameters must be of opposite signs, that is, if one is positive the other is negative. Indeed, as we shall see in section 3, the author generally assumes that permutation and translation work in opposite directions.

We note next that

$$ap + nt = a(p + kn) + n(t - ka).$$

In other words, adding to $p$ some multiple $k$ of $n$ is equivalent to adding to $t$ the same multiple $k$ of $a$, that is (as noted by the author on p. 42), translation by $k$ octaves. This observation allows one to conveniently restrict the $p$-values to the range strictly in between 0 and $n$ if $p$ is positive, or between $-n$ and 0 if $p$ is negative. Working the algebra together with our two earlier assumptions,

$$a > n \geq 2, \qquad -(a + n) < VLS < a + n,$$

we find that the $t$-values are similarly constrained to the range between $-a$ and 0 if $p$ is positive, or between 0 and $a$ if $p$ is negative. Indeed, the corresponding ranges $0 < p < n, -a < t < 0$, or $-n < p < 0, 0 < t < a$, are generally assumed in the book. A final useful restriction that I will use is to pairs $(p, t)$ that satisfy $-\frac{a}{2} < t \leq \frac{a}{2}$.[9] I shall refer henceforth to such pairs as "standard."

As we shall see in section 3, $|VLS| < a + n$ is in fact far too generous a relation in terms of capturing the types of voice leadings that interest the author the most. The author, we shall see, is most interested in voice leadings that are *maximally, but not trivially, balanced*, that is, voice leadings of which $|VLS|$ *is minimal*, exclusive of the case $VLS = p = t = 0$. But before we can turn to the question of representing geometrically voice leadings of this sort, we must

[9] That is, translation travels the shortest of the two possible distances, e.g., ascending 4th rather than descending 5th, which, together with permutation operating in the opposite direction, yields the same overall result. In case the notion "shortest distance" is inapplicable, as in translation of $\pm 6$ relative to the 12-scale, the ascending route is assumed.

consider two mutually exclusive algebraic scenarios that have crucial implications for this task.[10]

Under the first scenario $a$ and $n$ are coprime, that is, their *greatest common divisor*—the largest number that divides both—is 1; henceforth I will use the often-encountered notation $\gcd(a,n) = 1$ to express this relation. For example, in Example 1, $a = 7$ is a prime number and is thus coprime with (or relatively prime to) any number; similarly, in Example 2(c) we have $\gcd(12,7) = 1$.

If $\gcd(a,n) = 1$, Equation 1 defines a bijection between $\mathbb{Z}_{an}$ and $\mathbb{Z}_a \times \mathbb{Z}_n$.[11] It follows that $VLS = 0$ if, *and only if*, $(p,t) = (0,0)$—that is, the voice leading is balanced trivially.[12] Therefore, any maximally, but not trivially, balanced voice leading satisfies $|VLS| = 1$. Writing $(p^-,t^-)$ for the unique standard $(p,t)$ if $VLS = -1$, and $(p^+,t^+)$ if $VLS = 1$, we have:

$$(p^-,t^-) = -(p^+,t^+) = (-p^+,-t^+),$$

that is, the two "$p$-values" are inverses one of the other (relative to addition), and similarly, the two "$t$-values."

Our task, then, is to find, relative to the given coprime $a$ and $n$, the unique standard pairs $(p^\pm,t^\pm)$ satisfying

$$ap^\pm + nt^\pm = \pm 1.$$

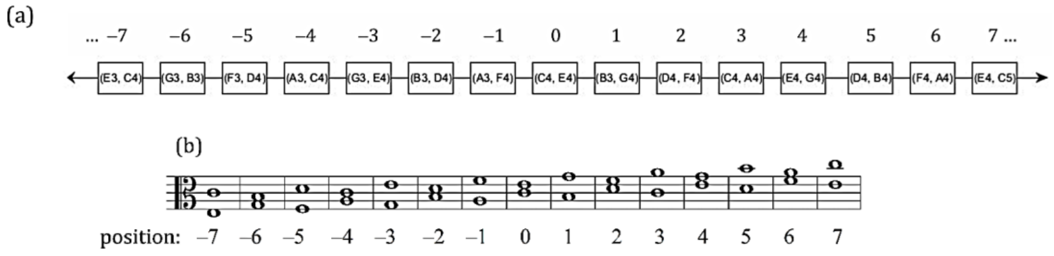
In general, an algorithm can be used to execute this task; but for small numbers, trial and error will usually suffice. For example, if, as in Example 1(a), $(a,n) = (7,2)$, then $(p^\pm,t^\pm) = (\pm 1,\mp 3)$.

We may now turn to the question that prompted this entire discussion. As an example of the case $\gcd(a,n) = 1$, how can one represent geometrically the recursive transformation of (say) the 7-scale dyad (C4, E4), by the two standard pairs of values $(p^+,t^+)$ and $(p^-,t^-)$ for permutation and translation each, namely $(1,-3)$ and $(-1,3)$, values that represent non-trivially maximally balanced voice leading?

The simplest representation would be a straight line extending indefinitely in both directions from a point labeled (C4, E4), such that the line is ruled from this zeroth point (or zeroth "position") in either direction into segments of equal length (Example 5(a)). Each segment-defining point on the line represents the transformation by permutation and translation of (C4, E4). Moving along the

[10] I apologize to readers that are less mathematically inclined, if the discussion that follows is mathematically a bit dense (and a bit lengthy as well). Examples 5–7 capture geometrically the main ideas discussed and should be readily accessible.
[11] This follows from the so-called "Chinese remainder theorem."
[12] Recall that standard pairs $(p,t)$ are assumed.

line to the right or positive direction, for any pair of adjacent dyads, the second dyad is the transformation of the first by $(p^+, t^+)$, $(1, -3)$ in our example, such that $VLS = 1$; moving to the left, the transformation is by $(p^-, t^-) = (-1, 3)$, and $VLS = -1$.[13] In Example 5(b), the dyads of Example 5(a), in a left to right order, are notated on the staff.



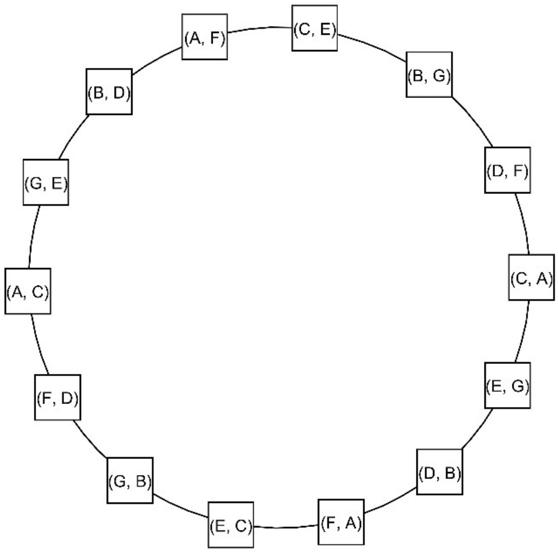

**Example 5** (a) Linear graphic representation of recursive permutation and translation of the 7-scale dyad (C4, E4), $VLS = \pm 1$, moving to the right (respectively, to the left). Positions along the line are numbered above the graph. (b) The dyads of Example 5a in staff notation

A possibly more useful representation, however, might make explicit the observation, that it takes exactly $an$ iterations (in our example, $an = 7 \cdot 2 = 14$), for the recursive process of permutation and translation to yield an octave translated $n$-ad: compare in Example 5 positions $-7$ and 7.[14] To represent this observation, the line may be "twisted" in three dimensions to form a spiral (or helix), such that the circumference of the spiral when "looked down" at, as it were, from "above," is $an$ segments long. As shown in Example 6, this three-dimensional representation may be collapsed into two dimensions, depicting exactly the "vantage point" just mentioned. Under this representation, rather than a line of indefinitely many segments, we have a circle the circumference of which is exactly $an$ segments long. Each of the $an$ positions on the circle represents an *octave class*, that is, a maximal class of $n$-ads all relatable to each other by translation by an arbitrary number of octaves. (For this reason, the fourteen "hours" in Example 6 are labeled with dyads that lack register designation.) Any single clockwise move represents the *class* of permutation and translation $(p^+ \bmod n, t^+ \bmod a)$, and the *class* of voice-leading sums $1 (\bmod\, an)$; for the counterclockwise direction $(p^-, t^-)$ replaces $(p^+, t^+)$, and

[13] There is no "meaning" to "right, positive, $(p^+, t^+)$," versus "left, negative, $(p^-, t^-)$," except that the two sets of terms refer to opposite directions. From the purely formal point of view, any two corresponding terms in these two triples are interchangeable.

[14] This result also follows from the relation $\gcd(a, n) = 1$.

the class of voice-leading sums is $-1 (\mathrm{mod}\, an)$.[15] So much for the case $\gcd(a, n) = 1$.




**Example 6** Circular graphic representation of recursive permutation and translation of the 7-scale dyad-class (C, E), $VLS \equiv \pm 1$ (mod 14), moving clockwise (respectively, counterclockwise)

If $a$ and $n$ are *not* coprime (that is, $\gcd(a, n) > 1$), then there are *no* solutions to Equation 1 for $VLS = \pm 1$, and *more than one solution* for $VLS = 0$. Therefore, our task under this scenario is to find (standard) solutions, other than $(p, t) = (0, 0)$ (the trivially balanced voice leading), to the equation

$$ap + nt = 0.$$

In general, as in, e.g., $(a, n) = (12, 8)$, we cannot assume that $n$ divides $a$ (8 does not divide 12, but has a divisor larger than 1, namely 4, that divides 12 as well). In the book, except for a passing reference in appendix 2 (see the rightmost diagram in Figure A2.12, p. 553), there is no example illustrating this general case.[16] Therefore, I will consider here only the special case where $n$ divides $a$, as in $(a, n) = (12, 3)$ (but see notes 17 and 18, and Example 2(b)).

If $n$ divides $a$, assume that the $n$-ad of interest is translated by some multiple $k$ of $\frac{a}{n}$.[17] For example, if $(a, n) = (12, 3)$, assume that the triad is

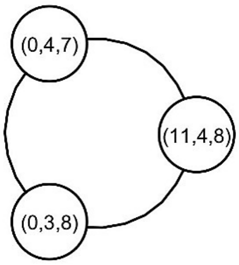
[15] Similarly to the line model (see n. 13), "clockwise" and "counterclockwise" may be interchanged at no cost.

[16] In the appendix reference on p. 553, the "size" of the scale (the number $a$ in this review) is not given, though it can be neither 7 nor 12, as the "size" of the chord (the number $n$ in this review) is apparently 4.

[17] More generally, $\frac{a}{g}$, where $g = \gcd(a, n) > 1$.

translated by some multiple $k$ of $\frac{12}{3} = 4$. Under this assumption, the formula $ap + nt$ assumes the form $a(p + k)$.[18] It is easy to see that for $a(p + k)$ to equal zero, $p$ and $k$ must be additive inverses one of the other. For example, again with $(a, n) = (12, 3)$, if $k = \pm 1$ (i.e., $t = \pm 4$), then $p = \mp 1$; if $k = \pm 2$ ($t = \pm 8$), $p = \mp 2$, etc. "Weeding out" the "non-standard" $(p, t)$ pairs, we are left, in addition to the trivial $(p, t) = (0, 0)$, with $(p, t) = (\pm 1, \mp 4)$. It is not difficult to see that these three are the only standard solutions of the equation $12p + 3t = 12(p + k) = 0$.

Example 7 is a geometric representation of non-trivially maximally balanced voice leading, $n$ divides $a$. The example assumes $(a, n) = (12, 3)$, with the triad of interest being $(0, 4, 7)$. Note that after $n - 1$ iterations (two, in the example), the recursive process cycles back to the exact point of origin. Accordingly, the diagram is a circle with a circumference consisting of exactly $n$ segments.



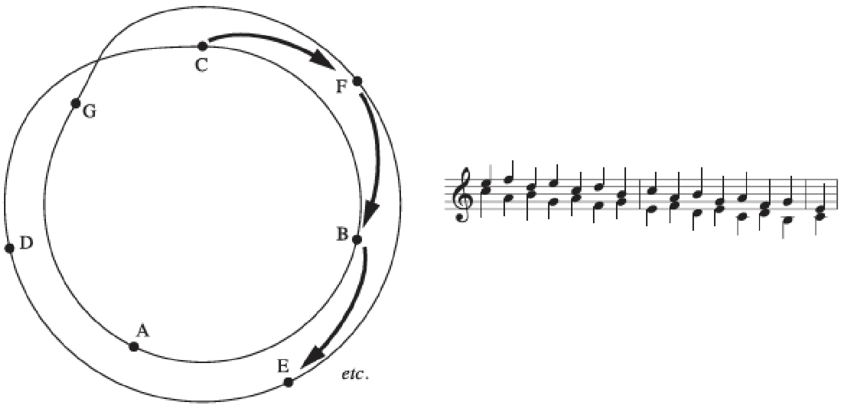

**Example 7** Circular graphic representation of recursive permutation and translation of the 12-scale triad (0, 4, 7), $VLS = 0, (p, t) \neq (0,0)$

It is important to note how different from each other are the superficially similar diagrams in Examples 6 and 7. Example 6, the reader will recall, is a two-dimensional circular projection of a *three-dimensional* spiral or helix that conceptually extends indefinitely in space. Under this projection, each position on the circle represents an infinitely large *class* of $n$-ads, namely, the maximal class of $n$-ads all relatable by (some) octave translation to one of its members. In Example 7, by contrast, the two-dimensional circle with its $n$ positions, each position representing exactly one $n$-ad, is all there is. Moreover, whereas in Example 6, all $a$ distinct translations modulo the octave translation, of any member of any given class, are represented, in Example 7 a representation exists for only a subset of exactly $n$ such translations (namely, the $n$ positions on the circle, 3 in our example).

[18] More generally, $a\left(p + k\frac{n}{g}\right)$, where $g = \gcd(a, n) > 1$.

# 3.

The "spiral diagrams" are described by the author as "the main theoretical models used in this book" (p. 580). In assessing their success as models, it will be useful to begin with the diagram of Figure 3.1.3 (p. 99), reproduced here as Example 8. Under the terms used in this review, the diagram assumes $(a, n) = (7, 2)$, the dyad of interest being of the type (C, E). All dyads in the diagram are represented by their "roots," i.e., "C" for (C, E), "F" for (F, A), etc.; as the author explains, "looping around the spiral"—by which he presumably means moving clockwise from "C" to "F" (for example), passing "B," "A," and "G"—"transposes along the chord, turning thirds into sixths [e.g., (F5, A5) into (A4, F5)] and vice versa" (p. 98).




Example 8 The book's Figure 3.1.3 (p. 99). "A series of clockwise moves in the space produces a descending-fifth progression in which thirds and sixths alternate." 

The first thing to note about Example 8 is that although the graph as such is drawn as a loop that winds around twice before returning to the point of origin, and moreover, consecutive positions *on the loop* are labeled with step-related dyads (roots C, B, A, etc., moving clockwise), the author's attention is rather focused on the *descending fifth relationships* that straddle the two strands of the loop (C to F, F to B, etc.): these relationships are not only singled out by the heavy curved arrows, but are spelled out in staff notation at the right-hand portion of the figure. Indeed, referring to Figure 3.1.2, a version of the same diagram on the previous page, the author singles out the voice leading associated with the *descending-fifth* root progression (as in C5 moving to A4 in the lower voice together with E5 to F5 in the upper, in the first two dyads given on the staff)—*not the voice leading associated with the descending-second root progression* (C5 to B4, E5 to D5)—as "the *descending basic voice leading* for diatonic thirds, since it connects every point on the graph to its nearest clockwise neighbor" (p. 98, original emphasis). As the author clarifies in a footnote, this particular voice leading is defined by a progression that "reduces the dyad's center of gravity by one" (*recte*, $\frac{1}{2}$, as the author in fact shows subsequently in the same footnote), a condition equivalent to $VLS = -1$.[19] "Repeatedly applying the basic voice leading," the author suggestively adds shortly thereafter, "moves through the two inversions … of the interval (third and sixth), touching on each inversion of every imperfect consonance before returning to the initial chord, now with the notes transposed by octave (Figure 3.1.3)."

Why the author draws the graph in a manner so blatantly at odds with what seems to be its main point will be conjectured subsequently in this section. For the moment, four comments are in order.

(1) It is contrary to accepted norms of graphing *not* to provide, a priori, edges connecting pairs of vertices (e.g., "C" and "F") that form a relationship regarding which the graph pretends to make some claim: cf. the claim on p. 98 that, for example, "F"—*not* "B"—is the "nearest clockwise neighbor" of "C."

(2) If "looping around the spiral" indeed means moving, e.g., from "C" to "F" by way of "B," "A," and "G," then the motion from "G" to "F" cannot be different in kind from the other stepwise motions, meaning that thirds remain thirds, and similarly sixths. Otherwise, "looping around the spiral" has a private meaning for the author, outside the realm of ordinary graph-theoretic discourse.[20]

(3) The "loop rule," in any case, works as expected only in one direction. For example, in Example 8 one must move *clockwise* along the spiral from "C" to "F," passing "B," "A," and "G," to (supposedly) represent the (non-standard) case $(p, t) = (1, -4)$, $VLS = -1$; the counterclockwise direction by way of "D" and "E" does *not* express the inverse case, $(p, t) = (-1, 4)$. It follows that prior, *non*-graphic knowledge is needed for the graph to be useful.

(4) Finally, and crucially, the graph fails to make sense as a representation of "hierarchically nested transposition" (p. 42), for the simple reason

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[19] As the author explains on p. 87, the "center of gravity" of an $n$-ad is the average $\frac{c_0+c_1+\cdots+c_{n-1}}{n}$ of its elements $c_i$. Thus, a *change* in the center of gravity is $\frac{VLS}{n}$. The author's bias towards negative voice-leading sums will be considered shortly.

[20] Such a private meaning is indeed suggested on p. 44, where the author explains that "a full *loop* around the center of the space … requires 'jumping rings,' or leaving the spiral at some point, as discussed in appendix 2" (original emphasis; see the upper portion of Figure P2.11, p. 45). However, in the appendix mentioned, in a context referred to in n. 21, the author correctly notes that "points not on either helix have no musical meaning" (p. 546).

> that each node, *by itself*, already represents "transposition along the collection" (that is, non-zero permutation): the node "C," for example, represents both (C5, E5) and (E4, C5), as the staff-notated portion of the figure makes explicit (compare the second measure with the first).[21]

If, however, one follows through twice the *descending-fifths* path traced by the heavy curved arrows, effectively ignoring the loop drawn, with clockwise and counterclockwise interchanged Example 8 may be construed as equivalent to Example 6, the two-dimensional circular projection of a (hypothetical) helical diagram illustrating the case $a$ and $n$ are coprime.

Regardless of whether Example 8 is but a poorly executed version of Example 6, an apparently important motivation of the author's for presenting it—the rationalization of (triadic) 7-scale root progressions by descending fifth—misses the mark, both theoretically and analytically. In section 4 I will consider the analytic perspective that concerns the root progression by fifth; here I will consider the theoretical perspective that concerns its descending direction.

As noted in section 2 (see notes 13 and 15), such terms as "positive," "ascending," and "clockwise" may be respectively interchanged, at no formal cost, with "negative," "descending," and "counterclockwise." Moreover, the author seems especially interested in *opposite-signed* $p$ and $t$, that is, in permutation (AKA "transposition along a collection") and translation that work in contrary directions, so that if one "descends," the other "ascends."[22] Thus, even if it were true, as the author claims on p. 47, that "in many musical styles, melodic steps tend to descend while leaps tend to ascend," his claim on p. 99 that "purely contrapuntal relationships [i.e., 7-scale two-voice configurations] will tend to privilege [descending] fifth motion," is not supported by the formalism used (or at any rate, implied), and even less so by the constraints under which the author seems particularly interested in engaging it.

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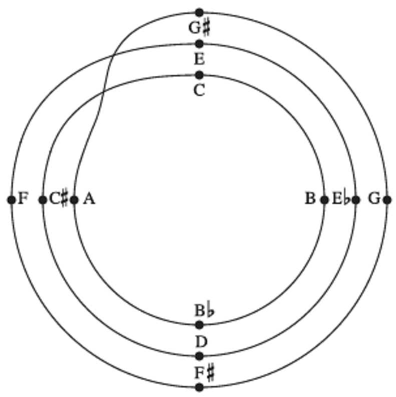
[21] In the book's appendix 2, pp. 545–46, the author employs the idea of a "union of helices" to generate such permutationally multivalent nodes.

[22] Numerous comments in the book strongly suggest that this is the case, starting with the following comment on p. 15: "Particularly important is the technique of counteracting, or nearly counteracting, an operation at one level with its analogue at another: combining transposition along chord [i.e., Tymoczkian permutation] and scale [i.e., translation, in a direction opposite to that of permutation] to produce efficient voice leading…." On the other hand, when the author seems to allow, as in the spiral diagrams, for such pairs $(p, t)$ as $(0, \pm 1)$, the result, as we have just seen, is incoherent.

Though the term "efficient voice leading" is not defined in the book, from the following statement on p. 88 we learn that the author believes that "maximal efficiency" (presumably, in some non-trivial sense) entails maximal balance: "the most efficient voice leading between two vertical configurations will necessarily minimize the change in center of gravity" (i.e., maximize the balance—see n. 19). Therefore, in the statement just quoted it seems safe to substitute "balanced voice leading" for "efficient voice leading."

The author's Figure 2.1.3 (p. 49), repeated numerous times in chapter 2 and reproduced here as Example 9, is the spiral diagram for "chromatic triads"; that is, $(a, n) = (12, 3)$, the triad of interest being of the type $(0, 4, 7)$. If Example 8 was the author's version of a case representing the scenario $a$ and $n$ are coprime, Example 9 is his version of the scenario $n$ divides $a$. As in Example 8, chords are represented by their "roots."




**Example 9** The book's Figure 2.1.3 (p. 49). "The spiral diagram for chromatic triads. Each point represents a complete major triad." © Oxford University Press 2023. Reproduced with permission of Oxford Publishing Limited through PLSclear

Example 9 inherits all the deficiencies noted in connection with Example 8. Briefly, edges expressing "*radial* motion between nearby chords" (p. 49, original emphasis), e.g., "C" to "E," are missing; "looping around the spiral" is not only ill-defined, but achieves the desired voice leading in only one direction (as the author acknowledges in note 3, p. 50); and crucially, "transposition along the collection" is pre-embodied in each node independently of any motion between nodes.

Strikingly missing in the author's discussion of Example 9, compared to Example 8 and other diagrams of the type $a$ and $n$ are coprime (notably Figure 3.4.1 on p. 113 for "diatonic triads"), is the notion of "basic voice leading." Ignoring the author's bias towards the negative direction, under the scenario $a$ and $n$ are coprime, we have seen, "basic voice leading" is defined by a change in the chord's "center of gravity" of $\frac{1}{n}$, a definition equivalent to $|VLS| = 1$. Under the scenario $n$ divides $a$, the analogous definition, equivalent to $VLS = 0$, $(p, t) \neq (0, 0)$, would have been: "unchanged center of gravity, non-trivially." The omission of "basic voice leading" in connection with Example 9 is particularly puzzling, since in reference to "radial motion," exemplifying precisely the idea of (non-trivially) unchanged center of gravity, the author makes some of the most suggestive comments in the book in terms of VLS/TTPC (p. 69, emphasis added):

> [The] basic principles [of the spiral model] are completely independent of a chord's specific intervallic content, depending only on the size of the chord and the size of the scale…. In each case [i.e., various triads relative to the 12-scale used by the author as examples], radial motion involves the same total amount of upward and downward semitonal motion, so that *the sum of the paths in all voices* is zero semitones. This universality *is one of the most remarkable features of the geometry*, a single $n$-in-**o** graph describing any $n$-note chord in any **o**-note scale.[23]

The probable reason for the author's omission of "basic voice leading" under the scenario $a$ and $n$ are *not* coprime, may also explain the idiosyncrasies of the spiral diagrams in general.

If $a$ and $n$ are not coprime, write $\gcd(a, n) = g > 1$. In such a case, the postulated "basic voice leading" (i.e., the voice leading associated with $VLS = 0$, $(p, t) \neq (0, 0)$) generally generates a set of exactly $g$ distinct translations, modulo the octave translation, of the $n$-ad of interest.[24] If $n$ divides $a$ we have $g = n$, so that the number of distinct translations is $n$, as we have seen in Example 7, where $(a, n) = (12, 3)$. In the general "recipe" given on pp. 42–43 for drawing spiral diagrams, the author derives rule A, "draw a spiral with $n$ loops, attaching its end to its beginning" (see Figure P2.9 on p. 43), by apparently generalizing from the case $n$ divides $a$. The author derives rule B, "mark off **o** [i.e., $a$] equally spaced points along the spiral, labeling them with consecutive scale tones" (see Figure P2.10 on p. 43), by apparently generalizing from the case $a$ and $n$ are coprime, where, as we have seen in Example 6, the "basic voice leading" generates all $a$ distinct translations of the $n$-ad (modulo the octave translation). Both apparent generalizations, possibly expressions of what the author refer to on p. 5 as "the Prime Directive" (see the discussion of "Gesualdo's trick" in the following section of this review), are not supported by the underlying algebra.

## 4.

I begin this more analytically oriented section of the review with two debts: a debt from the previous section regarding the author's bias, most notably in a 7-scale dyadic context, for $VLS = -1$, and a debt from section 2 regarding "Tymoczkian permutation," AKA "transposition along a collection." I then turn to the author's claims regarding the opening of Gesualdo's madrigal *Moro lasso*, the first music analyzed in the book.

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[23] Note that "the sum of the paths in all voices" is $VLS$. Supplying references that provide no supportive argument, the author states on p. 575 that a basic voice leading "is found only when the size of the chord is relatively prime to the size of the scale (§3.1, §3.4, appendices 1–2)."

[24] The qualification "generally" is meant to exclude "symmetrical" $n$-ads, such as the 12-scale "octatonic collection."

On p. 99, the author claims vis-à-vis Example 8 that "purely contrapuntal relationships will tend to privilege [descending] fifth motion: if we are in two-note diatonic space, then fifth-related dyads are literally adjacent to one another." In the previous section I commented on the lack of formal support for the author's bias towards 7-scale dyadic root motion by descending rather than ascending fifth (that is, for $VLS = -1$ rather than $VLS = 1$), a bias so strong that the qualification "descending" in the statement just quoted is allowed to be understood implicitly. In support of this statement, the author proceeds to present excerpts in two voices from Palestrina and Beethoven. Regarding the Beethoven, the opening measures of the last movement of the Piano Sonata in E-flat major, Op. 27, No. 1 (Figure 3.1.6 on p. 100), if only by placing Roman numerals underneath the excerpt's two-voice reduction, the author acknowledges that the dyads represent triads. I would argue, however, that a triadic structure is implicit also in the two-voice opening of Palestrina's mass *Ave regina coelorum* (Figure 3.1.5 on p. 100), as rendered explicit in Example 10 by unstemmed black noteheads and dashed slurs. To be sure, the stylistic norms to which Palestrina seems to have felt obliged would have prevented him from *expressing* the root-position diminished triad (B, D, F) of m. 3 in three voices; but such is the relationship, in music, between "surface" and "deep structure."


**Example 10** (after Figure 3.1.5 on p. 100). Implicit triadic structure in the two-voice opening of Palestrina's mass, *Ave regina coelorum*. Unstemmed black noteheads represent implied notes

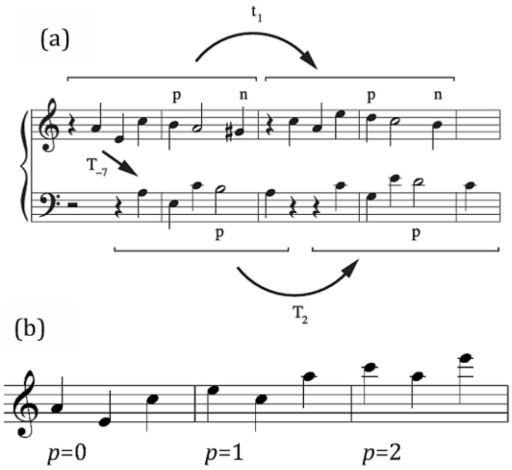
It is important to stress the underlying triadic structure of both the Beethoven and Palestrina excerpts, since, as the author discusses on p. 113, in the case of "diatonic triads," $(a, n) = (7, 3)$, the root progression that expresses the "basic descending voice leading" $VLS = -1$, is by *ascending third* rather than the presumably desired descending fifth. Thus, an apparent motivation for the author's dyadic Example 8 is to make a case for *triadic* progression by descending fifth—inappropriately, as I have argued theoretically (in the previous section) regarding the descending direction and analytically (right now) regarding the fifth.[25]

[25] By "analyzing triadic music using *dyadic* logic" (p. 9, original emphasis), the author refers precisely to what I have just argued is an analytical *mis*appropriation of Example 8 to excerpts like the Palestrina and Beethoven.

In section 2 I have pointed out that the author's idea of "transposition along a collection"—"Tymoczkian permutation," as I prefer to refer to it—is a generalization of the elementary-theory concept of "inversion." Though valuable, the idea falls short of being "the single most important concept in music theory" (p. 37). In the context of VLS/TTPC, for example, the "cost" of assuming traditional inversion in place of Tymoczkian permutation is not that unbearable: restricting the discussion to chords in closed position, ordered from low to high.

Consider in this connection the imitative mm. 3–6 of Domenico Scarlatti's Sonata in A minor, K.3, a passage that the author analyzes in terms of the "purely diatonic" 7-scale in the upper portion of Figure P2.3 on p. 39, reproduced as Example 11(a). The main point of the analysis is to show that, in addition to the descending octave translation from right hand to left (see the straight arrow labeled $T_{-7}$ between the staves), the bracketed opening idea is translated ("transposed along the scale") in the left hand by ascending 3rd ($T_2$, equivalent to $t = 2$ in this review—see the curved arrow below the staves), and "transposed along the chord" (that is, permutated) in the right hand ($t_1$, equivalent to $p = 1$ in this review—the curved arrow above the staves).[26] In other words, ignoring the return to A4 in m. 4 and the corresponding return in m. 6 to C5, the chord (A4, E4, C5) of m. 3 is presumably permutated once upward to (C5, A4, E5) in m. 5. However, (C5, A4, E5) is *not* the first upward permutation of (A4, E4, C5); in fact, it is not a permutation of (A4, E4, C5) at all. The two upward permutations of (A4, E4, C5) are shown in Example 11(b). As may be seen, (C5, A4, E5) is at best an octave translation of the original chord's *second* permutation.

[26] The "nonharmonic" B4 and G♯4 of m. 4 are ignored. See, in this connection, chapter 5, "Nonharmonic Tones" (pp. 210–52).




**Example 11** (a) From the book's Figure P2.3 (p. 39). "Domenico Scarlatti's Sonata in A minor, K.3, mm. 3–6." © Oxford University Press 2023. Reproduced with permission of Oxford Publishing Limited through PLSclear. (b) The two upward permutations of the chord (A4, E4, C5) of m. 3

That the two upward, Tymoczkian permutations of (A4, E4, C5), are indeed as depicted in Example 11(b) may seem puzzling at first. And yet, the first upward permutation of (A4, E4, C5), a chord representing the "klang" (A, E, C),[27] begins, as shown in Example 11(b), with E—not C. For, when "we compress … [some] chord's notes into a single octave" (p. 38), *unless the chord represents a klang of a special type* to be specified shortly, we may implicitly alter the cyclic ordering of the klang represented. For example, when Scarlatti's (A4, E4, C5), representing (A, E, C), is "compressed" into the octave above and including A4, ordered from low to high the chord becomes a representative of (A, C, E), thus reversing the cyclic order of E and C. Only if the chord, as defined in the appendix of this review, represents a *standard* klang such as (A, C, E),[28] does its "compression" into a single octave preserve the original cyclic order. For example, (A4, C5, E5), (A4, C6, E5), and (A4, C5, E4), all represent the standard klang (A, C, E).[29] In short, the Scarlatti example does *not* represent, as the author believes, Tymoczkian permutation, AKA "transposition

[27] A "klang" is an ordered set of step-*classes*, where "step class" is an equivalence class containing all and only steps, octave-related to one of the members of the class. See Definition 1.1 in the appendix to this review.

[28] Klangs such as (A, C, E) or (A, C, E, G), in any cyclic permutation, are standard. See Definition 3.1 in the appendix to this review. The klang (A, E, C) is *not* standard.

[29] Equation 1 is proven in the appendix of this review relative to standard klangs. Note that a chord such as (A4, C5, E4) need not necessarily be expressed *melodically*, that is, ordered in time. For example, what is sometimes referred to as a "consonant six-four chord" of the "waltz or march type" (Edward Aldwell and Carl Schachter, *Harmony and Voice Leading*, 2nd ed. (San Diego: Harcourt Brace Jovanovich, 1989), 300), may be thought of as a chord exactly like (A4, C5, E4), that is, a chord representing a root-position klang such that the representative of the fifth is placed below that of the root. In such a case, the vertically expressed closed-position chord is *not* ordered from low to high.

along a collection," but rather what may be described informally as an "arpeggiating sequence": a sequence the overall melodic progression of which outlines a chord.[30]

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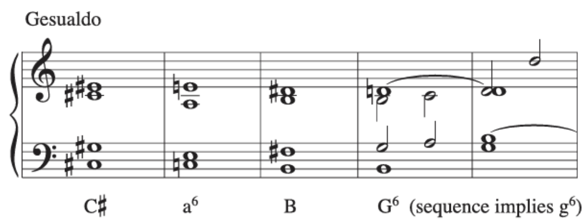
Following some introductory remarks on pp. 1–4, the book proper begins on p. 4 with the author's analysis of the famous opening progression of Gesualdo's madrigal *Moro lasso* (see Example 12, reproducing the upper portion of Figure 1.1.1, p. 5). According to the author, these measures exhibit a "trick" found also in Beethoven, Schubert, and Debussy (Figure 1.1.2), as well Mozart (Figure 1.1.3), Stravinsky (Figures 1.1.4 and 1.1.7), and Rudresh Mahanthappa (Figure 1.1.5).




**Example 12** From the book's Figure 1.1.1 (p. 5). "The opening of Gesualdo's 'Moro lasso' (1611)." © Oxford University Press 2023. Reproduced with permission of Oxford Publishing Limited through PLSclear

The motivation for opening the book with "Gesualdo's trick," the essence of which will be stated shortly, seems to be to introduce what for the author is "the Prime Directive: *whenever you find an interesting musical technique, try to generalize it to every possible chordal and scalar environment*" (p. 5, original emphasis). But I believe this is just an obfuscation. For, what is striking about the 1611 excerpt is not just the daring chromaticism couched in relatively smooth voice leading (even smoother if the passage is reduced to three voices, eliminating doublings), but that it systematically alternates root-position *major* triads with first-inversion *minor* ones: twice in a row, if one accepts the author's claim that the G-major sixth-chord replaces an implied G-minor one. As the reader will recall, the 7-scale obliterates the distinction between major and minor altogether (for this reason, the author's Figure 1.1.6, the "Pachelbel sequence" conceived in terms of the 7-scale, is irrelevant in this context); as to the 12-scale, since the VLS/TTPC insight, as noted in section 2, applies only to

[30] Possibly akin to what the author refers to on p. 188 as "structured arpeggiation." Had fugal answers *in minor* been oriented towards the mediant key rather than that of the (minor) dominant, Scarlatti's left-hand arpeggiating sequence would have represented a "real" answer, whereas that of the right hand, a "tonal" one.

chords of a given type (that is, chords related by permutation and translation), it has nothing to say regarding passages like the Gesualdo, that mix triads of such different types as (0, 4, 7) and (0, 3, 7).[31] The hidden motivation for opening the book with "Gesualdo's trick," therefore, is to work around this critical theoretical impasse.

The author does not address the impasse explicitly until section 6 (pp. 68−71) of chapter 2, "Rock Logic" (pp. 47−95). The chapter, for which the main theoretic model is the "spiral diagram for chromatic [major] triads" discussed earlier in this review as Example 9, analyzes, at least until section 7, musical excerpts consisting predominantly of major triads. "By now" the author appropriately notes on p. 68, "readers will be wondering how to fit minor triads into the spiral model."

The author's "first answer" to this question is that "Figure 2.1.3 [i.e., Example 9] can be used to represent *any* type of three-note chord in chromatic space" (p. 68, original emphasis). Only after a lengthy digression that drives home this point,[32] does the author acknowledge its irrelevance to the question asked: "of course, we would really like to represent both major and minor chords *at the same time*" (p. 69, emphasis added). He then proceeds to offer two solutions.[33]

The first proposed solution, illustrated in Figure 2.6.2 (p. 70), is to superimpose "major and minor spiral diagrams" such that the "minor diagram" is slightly rotated clockwise relative to the "major diagram."[34] Referring to "Gesualdo's trick," the author comments (ibid.): "we can use this graph to represent chromatic triadic sequences such as those in Gesualdo and Mozart (§1.1)." The second proposed solution is "to use Figure 2.1.3 [i.e., Figure 9] to represent major *or* minor triads, with the single point 'C' standing for both C major and C minor" (ibid., original emphasis).

It is not difficult to see that either "solution" merely compounds the severe problems already inherent in Example 9, as discussed in section 3 of this review. For example, under the first "solution," in addition to the conspicuously missing "radial" edges within *each* of the two superimposed triadic graphs, e.g., "C" to "E" in major and "c" to "e" in minor, edges are now missing as well between points lying one in each graph, e.g., "C" to "c." The second "solution,"

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[31] Tentatively assuming, that the 12-scale triads (0, 4, 7) and (0, 3, 7) are the same objects as the tonal "major" and "minor" triads, respectively. See section 5.

[32] As noted in section 3 of this review, the digression in fact contains some of the most suggestive comments in the book regarding the case $n$ divides $a$.

[33] A "third strategy for simultaneously modeling major and minor triads," suggested at the very end of this section (p. 71), is to use the 7-scale. As noted, this is equivalent to obliterating a priori the distinction between major and minor.

[34] In the printed version of the book, spiral diagrams for tetrads, not triads, are mistakenly superimposed at this point.

similarly, compounds the crucial problem of multivalent nodes. Indeed, when the author concludes this entire discussion with the statement "in the rest of the chapter, I will mostly follow the strategy of grouping [parallel] major and minor together, *sacrificing detail* in the name of graphical simplicity" (p. 71, emphasis added), he admits de facto of failing to have solved the problem.

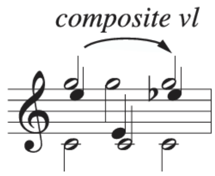
Where does all this leave us regarding "Gesualdo's trick"? To answer this question, I finally note that the "trick" is to have "two voices moving in parallel [e.g., soprano and bass in Example 12] with the third [e.g., the composite of tenor alternating with alto] *alternating between prime and inverted forms of a sonority*" (p. 6, emphasis added). The crux of the "trick," in other words, is to regard the "major" and "minor" 12-scale triads *as related by inversion*, in the mathematical sense that one is a reflection of the other.[35] Since VLS/TTPC is silent regarding "inversion" in this sense, the "trick" is thus a cover for the author's unacknowledged *first* attempt to solve the "major/minor problem."

The idea that VLS/TTPC may (conceivably) be expanded to incorporate both reflection and counter-permutation (that is, the *reversal* of cyclic order), is hinted already on p. 15, where the author notes (original italics, bold mine):

> Particularly important is the technique of counteracting, or nearly counteracting, an operation at one level with its analogue at another: combining transposition along chord [i.e., Tymoczkian permutation] and scale [i.e., translation, in a direction opposite to permutation] to produce efficient voice leading, **or combining *inversion* along both chord** [i.e., *counter* Tymoczkian permutation] **and scale** [i.e., reflection] **to produce Gesualdo's trick** (appendix 3).

However, only in appendix 1 (and again in appendix 3)—that is, more than 520 pages after the reader was introduced, under a false pretense, to "Gesualdo's trick"—the idea is finally made explicit. As may be seen in Example 13, reproducing part of Figure A1.1 (p. 534), the 12-scale chord ("C4," "E5," "G5") is first *counter*-permutated (that is, its cyclic order is reversed—note the reversed stems) to ("G5, "E4," "C4"), which is then reflected to become ("C4," "E♭5," "G5"). "The result" the author states on p. 535 "is a voice leading between two similarly voiced, inversionally related pitch-class sets, known to theorists as 'neo-Riemannian transformations.'" The author then claims, misleadingly: "These [neo-Riemannian] transformations are generalized as 'Gesualdo's trick' in §1.1."

[35] Note the distinction between "inversion" *as reflection* and "inversion" *as permutation*. The author clarifies the terminological ambiguity on p. 577.




**Example 13** From the book's Figure A1.1 (p. 534). "Inversion occurring at both the triadic and chromatic levels." © Oxford University Press 2023. Reproduced with permission of Oxford Publishing Limited through PLSclear

For, as the author sees it, neo-Riemannian transformations do not map (0, 4, 7) onto (0, 3, 7), but rather onto (7, 3, 0), that is, "root" and "fifth" are reflectively interchanged. Indeed, it is precisely this "dualistic" stance by which the minor (respectively, major) triad is a major (respectively, minor) triad "standing on its head," that forces the author to perform "inversion of inversion" (the only trick that I can see here), as he admits on p. 577 under "Terms, Symbols, and Abbreviations" (emphases added):

> Neo-Riemannian progression. A spacing-preserving voice leading *between inversionally related chords*; it can be decomposed into *a pair of inversions* along intrinsic and extrinsic scales…. Any such progression will preserve the distance between at least two voices *and can be used in Gesualdo's trick*.

## 5.

The noun "tonality" is conspicuously absent from the book's index, nor does it appear among the terms, symbols, and abbreviations explained on pp. 575–81. For the adjectival form, however, we find the following (580):

> Tonal. An ambiguous term that can refer to music that is functionally tonal (see above), or non-atonal. In addition, the adjective sometimes refers to keys and modes ("tonal center," "tonal region," "tonal plan"). I typically use the term as a synonym for "non-atonal," encompassing techniques common to a range of modal and functionally tonal styles.

Whatever one makes of this statement, it at least seems to suggest a dichotomy of "tonal" versus "atonal." But then, we read on p. 2 (emphasis added):

> To call … [the book's program] a "generalized tonal theory" would be a misnomer, for its techniques are broad enough to embrace extremes of consonance and dissonance, encompassing *both tonality and atonality*.[36]

[36] A search for "tonality" in the digital edition of the book reveals that this is practically the only stand-alone appearance of the term, its other appearances being almost exclusively part of the composite "functional tonality" (see n. 38).

Now, the author is totally within his rights to let the words “tonal” and “tonality” mean anything or nothing as he deems fit. But then, the author is *not* within his rights, not having provided coherent guidance as to what the first word in the title of his book means, to proceed and simply apply wholesale the *implicit* meaning of “tonality,” whenever his theory, whatever it is about, fails to deliver the goods. As “owners” of tonality,[37] we seem to have an intuitive sense of what the term includes, which is first and foremost an underlying “scale” that consists of *two* types of “steps,” namely “whole steps” and “half steps” (and hence, apropos Gesualdo, two types of “consonant triads,” namely “major” and “minor”), a scale that, moreover, can be “chromatically altered” *without compromising its basic identity* (as in certain usages, apropos Gesualdo again, of *musica ficta*, or the three traditional varieties of the minor mode, or, more generally, “modal mixture”); surely, “tonality” also includes such notions as “tonic” (and hence, “key”) and “functional harmony,” relevant especially (but arguably not exclusively) to music of the “common-practice” period.

Consider again the “spiral diagram for chromatic [major] triads” (Example 9 above), of which the “straightforward features of musical geometry” are supposedly reflected in “a variety of rock progressions” (p. 33), the topic of chapter 2. Leaving aside the severe logical deficiencies noted in section 3 that render questionable the diagram’s fitness to serve as a model of anything, or its inability, in any case, to address progressions that mix different types of chords as discussed in section 4, the diagram assumes, with no theoretical argument, that a 12-scale triad such as (0, 4, 7) is the same as the *tonal* triad (C4, E4, G4)—*not*, e.g., (B♯4, F♭4, G4). Subsequently in the same chapter, again with no theoretical argument, the author “rewrites the spiral diagram *using Roman numerals*” (p. 56, emphasis added), a form retained through the remainder of the chapter. Thus, a 12-scale triad of type (0, 4, 7), already tacitly assumed to be the same as a major triad in the tonal sense, is further assumed to be *the tonic*, no less. Any “rock logic” that this chapter may exhibit therefore results from precisely those attributes of tonality that the author, at least in the first half of the book, feels free to assume but fails to acknowledge, let alone address.[38]

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[37] “We speak of taking ownership of our own words” the author notes on p. 4, “and in our society people sometimes come to own objects of significant public concern—a historic home, a well-known work of art, an ecologically significant wetland, or a beloved sports team. In these cases, to be an owner is to be a custodian, temporarily inhabiting an office with responsibilities and duties that we might not have chosen.”

[38] As we have seen, the most that the author is willing to acknowledge with regards to tonality is that “tonal” “*can* refer to music that is functionally tonal” (p. 580, emphasis added), that is, music that uses a harmonic language “sometimes called ‘tonal’ as opposed to ‘modal’” (p. 576). Much of the latter part of the book, starting with chapter 6, “The Origins of Functional Harmony,” is about “tonality” in what seems to be a modally expanded sense of “functional tonality.” As noted at the outset of this review, I defer discussion of this part of the book to a sequel.

We are left to conclude that the entire first half of the book on which this review has focused is not about tonality in any normative sense of the term; alternatively, this substantial portion of the book is about “tonality” in a sense known only to its author.

# APPENDIX

## VOICE-LEADING SUMS UNDER TRANSLATION AND TYMOCZKIAN PERMUTATION OF CHORDS

### 1. Klangs and their Cyclic Permutations

DEFINITION 1.1. Fix $a > n \geq 2,\ a, n \in \mathbb{Z}$.
We shall refer to an ordered set $K$ of $n$ integer classes (mod $a$), as *a klang*.

DEFINITION 1.2. Let $K = (\kappa_0, \kappa_1, \ldots, \kappa_{n-1}), K' = (\kappa'_0, \kappa'_1, \ldots, \kappa'_{n-1})$, be klangs.
We shall write $K' = \mathrm{P}(K)$ if $\kappa'_i = \kappa_{i+1\ (\mathrm{mod}\ n)}$.

Note:

- P is a bijection on the set of klangs, hence the inverse function $\mathrm{P}^{-1}$ is defined.
- The group $(\mathrm{P}^q, \circ)$ of operations on $\{K\}$, $\mathrm{P}^q = \overbrace{\mathrm{P} \circ \ldots \circ \mathrm{P}}^{q \text{ times}}$ if $q \geq 1$, $\mathrm{P}^q = \overbrace{\mathrm{P}^{-1} \circ \ldots \circ \mathrm{P}^{-1}}^{|q| \text{ times}}$ if $q \leq -1$, is a group of cyclic permutations.

### 2. Chords, their Translations and “Tymoczkian Permutations”

DEFINITION 2.1. Let $K = (\kappa_i)$ be a klang.
We shall refer to an ordered set $C = (c_i)$ of $n$ integers as a *chord representing* $K$, if $c_i \equiv \kappa_i$ (mod $a$).

DEFINITION 2.2. Let $C = (c_0, c_1, \ldots, c_{n-1})$, $C' = (c'_0, c'_1, \ldots, c'_{n-1})$, be chords representing the klangs $K$, $K'$, respectively. We shall write:

(A) $C' = \mathbb{T}(C)$ if $c'_i = c_i + 1$.
(B) $C' = \mathbb{P}(C)$ if both
  (1) $K' = \mathrm{P}(K)$ and
  (2) $c_i < c'_i < c_i + a$.

Note:

- $\mathbb{T}$ and $\mathbb{P}$ are bijections on the set of chords, hence the inverse functions $\mathbb{T}^{-1}$ and $\mathbb{P}^{-1}$ are defined.
- $\mathbb{T}^t(\mathbb{P}^p(C)) = \mathbb{P}^p(\mathbb{T}^t(C)) = \mathbb{T}^t \circ \mathbb{P}^p(C)$, where $\mathbb{T}^t = \overbrace{\mathbb{T} \circ \ldots \circ \mathbb{T}}^{t \text{ times}}$ if $t \geq 1$, $\mathbb{T}^t = \overbrace{\mathbb{T}^{-1} \circ \ldots \circ \mathbb{T}^{-1}}^{|t| \text{ times}}$ if $t \leq -1$, and similarly for $\mathbb{P}^p$.
- The group $(\mathbb{T}^t,\circ)$ of operations on $\{C\}$ is a group of translations; the group $(\mathbb{P}^p,\circ)$ is termed in this review "Tymoczkian permutations."*

### 3. Standard Klangs and Voice-Leading Sums

DEFINITION 3.1. We shall refer to a klang $K = (\kappa_i)$ as *standard* if there exist representatives $\bar{\kappa}_i$ of $\kappa_i$ such that

$$\bar{\kappa}_0 < \bar{\kappa}_1 < \cdots < \bar{\kappa}_{n-1} < \bar{\kappa}_0 + a.$$

DEFINITION 3.2. Let $C = (c_0, c_1, \ldots, c_{n-1})$ and $C' = (c'_0, c'_1, \ldots, c'_{n-1})$ be chords. We shall write

$$\sum_{i=0}^{n-1} (c'_i - c_i) = VLS$$

and shall refer to *VLS* as the voice-leading sum from $C$ to $C'$.

THEOREM 3.1. Let $K$ be a klang.
Then the following are equivalent:

(1) $K$ is standard.
(2) For any chord $C$ representing $K$, and for any $p$ and $t$, letting $C' = \mathbb{P}^p \circ \mathbb{T}^t(C)$, we have:

$$VLS = ap + nt.$$

* $(\mathbb{P}^p,\circ)$ is not truly a group of permutations, hence "Tymoczkian permutations" is, strictly speaking, a misnomer. But since these operations are partially determined by the cyclic permutations $\mathrm{P}^q$ (see Definition 2.2(B)), the terminological transgression, I hope, may be excused.

*Proof*. Since the effect of arbitrary $t$ on *VLS* is obvious, we shall only consider the effect of $p$. Moreover, it suffices to treat $p = 1$, since the rest will follow by induction. That is, we only need to prove that if $C' = \mathbb{P}^1(C)$, then $K$ is standard if, and only if, $VLS = a$.

Denote $\bar{c}_i$ the representative of $c_i$ in the interval $[0, a)$, that is, $c_i = \bar{c}_i + m_i a$ for some integer $m_i$, $0 \le \bar{c}_i \le a - 1$. Using Definition 2.2(B) we have:

$$c_i' = \begin{cases} \bar{c}_{i+1} + m_i a \text{ if } \bar{c}_i < \bar{c}_{i+1}, \\ \bar{c}_{i+1} + (m_i + 1)a \text{ if } \bar{c}_{i+1} < \bar{c}_i. \end{cases}$$

The number of indices $i$ (mod $n$) such that $\bar{c}_{i+1} < \bar{c}_i$ is at least 1, and is exactly 1 if and only if there exists an index $j$ such that $\bar{c}_j < \bar{c}_{j+1} < \cdots < \bar{c}_{n-1} < \bar{c}_0 < \cdots < \bar{c}_{j-1}$. This last condition is easily seen to be equivalent to the condition that $K$ is standard. This proves the theorem.